Part 1: Discrete Mathematics (DM) - Solve \((DM1 \land (DM2 \lor DM3))\)

**Question DM1**

25 points

A- Let \(P(n)\) be the statement that a postage of \(n\) cents can be formed using just 4-cent stamps and 7-cent stamps. Prove using strong induction that \(P(n)\) is true for \(n \geq 18\).

B- Let \(m\) be an integer with \(m > 1\). Show that the relation \(R = \{(a, b) \mid a \equiv b \pmod{m}\}\) is an equivalence relation on the set of integers. Then, determine what are the equivalent classes of the following relations:

a) \(R_1 = \{(a, b) : a \equiv b \pmod{7}, a, b \in \text{Primes}^+\}\)

b) \(R_2 = \{(a, b) : a \equiv b \pmod{7}, a, b \in \mathbb{Z}\}\)

c) \(R_3 = \{(a, b) : a \equiv b \pmod{7}, a, b \in \{1, 2, \ldots, 5\}\}\)

d) \(R_4 = \{(a, b) : a \equiv b \pmod{7}, a, b \in \text{even}\}\)

C- Let \(R_1\) and \(R_2\) be relations on a set \(A\) represented by the matrices

\[
M_{R_1} = \begin{bmatrix}
0 & 1 & 0 \\
1 & 1 & 1 \\
1 & 0 & 0 \\
\end{bmatrix}
\quad \text{and} \quad
M_{R_2} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

Find the matrices that represent \(R_1 \cup R_2\, R_1 \cap R_2\, R_2 \circ R_1\, R_1 \circ R_1\, R_1 \oplus R_2\, R^{-1}_1\).

Find the symmetric closure of \(R_1\), reflexive closure of \(R_1\), and transitive closure of \(R^{-1}_1\).

**Question DM2**

25 points

A- Suppose that \(A = \begin{bmatrix}
a & 0 & 0 \\
0 & b & 0 \\
0 & 0 & c \\
\end{bmatrix}\) where \(a, b, c \in \mathbb{R}\), prove that \(A^n = \begin{bmatrix}
a^n & 0 & 0 \\
0 & b^n & 0 \\
0 & 0 & c^n \\
\end{bmatrix}\)

for every positive integer \(n\) [using mathematical induction].

B- Find the integer \(a\) such that \(a \equiv -11 \pmod{21}\) and \(80 \leq a \leq 100\).

C- Find the values of the following:

a) \(\gcd(1000, 5050)\)  

b) \(\text{lcm}(1000, 5050)\) [use prime factorization b&c]

D- Determine whether each of these conditional statements is true or false.

a) If cat can fly, then you can pass MATH 3 exam  
b) If cat cannot fly, then you can fly  
c) If you can fly, then the cat can fly  
d) If you can fly, then \(25+25+25+25 = 0\)

**Question DM3**

25 points

A- Let \(P(m, n) : \text{"m divides n," where the domain for both variables consists of all positive integers. Determine the truth values of each of these statements.}\)

a) \(P(4, 5)\)  
b) \(P(2, 4)\)  
c) \(\forall m \forall n \ P(m, n)\)  
d) \(\exists m \forall n \ P(m, n)\)  
e) \(\exists n \forall m \ P(m, n)\)  
f) \(\forall P(1, n)\)

B- Prove that if \(m + n\) and \(n + p\) are even integers, then \(m + p\) is even, where \(m, n, p\) are integers.

C- Show that \(p \leftrightarrow q\) and \((p \land q) \lor (\neg p \land \neg q)\) are logically equivalent by developing a series of logically equivalences.

D- Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) \(\forall x \forall y ((x^2 = y^2) \rightarrow (x = y))\)  
b) \(\forall x \exists y (y^2 = x)\)  
c) \(\forall x \forall y (xy \geq x)\)
Part 2: Linear Algebra (LA) - Solve (LA1 ∨ (LA2 ∨ LA3))

**Question LA1**

25 points

A- Find the parametric form of the general solution of the system whose augmented matrix is given by

\[
A = \begin{bmatrix}
1 & 0 & -5 & 0 & -8 & 3 \\
0 & 1 & 4 & -1 & 0 & 6 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Find the bases of \( \text{Col } A \), \( \text{Row } A \), and \( \text{Nul } A \), then find the rank \( A \), dim \( \text{Nul } A \).

B- Let \( A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \end{bmatrix} \) and \( B = \begin{bmatrix} -3 & 5 \\ 1 & 2 \end{bmatrix} \), how to compute \( A^{-1}B \) without computing \( A^{-1} \). [Idea only]

C- Given \( A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \) where \( A^2 = I \), show that \( M^2 = I \), where \( M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \) [matrix partitions]

**Question LA2**

25 points

A- Given det \( \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7 \), find the determinants \( \begin{vmatrix} 5g & 5h & 5i \\ 2d + a & 2e + b & 2f + c \\ a & b & c \end{vmatrix} \).

B- Given subspace \( H \& M \) of a vector space \( V \), if \( H + M = \{ w: w = u + v; \ u \in H \} \wedge \{ v \in M \} \), show that

1) \( H+M \) is a subspace of \( V \).
2) \( H \) is a subspace of \( H+M \).

C- Let \( u = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \), \( v = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \), show that \( n \) is in \( \text{span} \{u, v\} \) for all real numbers \( n \) and \( k \).

D- Given \( p_1 = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} \), \( p_2 = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} \), \( p_3 = \begin{bmatrix} 3 \\ -4 \\ -7 \end{bmatrix} \) find the following: length of \( p_1 \); unit vector of \( p_2 \);

distance between \( p_1 \) and \( p_2 \), finally determine whether the set \( \{p_1, p_2, p_3, p_4\} \) is orthogonal set.

**Question LA3**

25 points

A- Diagonalize the following matrix \( A = \begin{bmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \), then find \( A^{50} \) if possible.

[show the following: characteristic equation, eigenvalues, bases of eigenspaces, matrix \( D \), matrix \( P \)]

B- Find the matrix of quadratic form, assume \( x \in \mathbb{R}^5 \) and \( 4x_1^2 + 4x_2^2 - 3x_3x_4 + 2x_4^2 + 3x_1x_2 - 5x_3x_4 - 4x_1x_4 \).

C- Let \( H \) be the set of points inside and on the unit circle in the xy-plane, i.e. \( H = \{x: x^2 + y^2 \leq 1\} \), show that \( H \) is not a subspace of \( \mathbb{R}^2 \).

D- If \( v_1, v_2, v_3, \) and \( v_4 \) are in \( \mathbb{R}^4 \) and \( v_3 = 2v_1 + v_2 \) then \( \{v_1, v_2, v_3, v_4\} \) is a linearly independent set.

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**Good Luck**
Answer the following three questions: (The total marks: 70)

1st Question marks: 25

a) Analyze the following paragraph then answer the following questions:

There are many differences between the geography of Ancient Greece and Ancient Egypt. Although Ancient Greece was mountainous, Ancient Egypt has the Nile River. Because of this, Ancient Greece had city-states. On the other hand, Ancient Egypt was united. In addition, Ancient Greece had less than ¼ of land for farming. However, Egypt could farm all over the Nile River. Even so, Greece grew olive trees for oil and other uses. Whereas, Egypt grew wheat and papyrus plants. In conclusion, the two civilizations differed in their geography.

i) Highlight the topic sentence and the concluding sentence. (4 Marks)

ii) Mention the type of the paragraph (comparison, contrast, cause and effect, persuasive). (2 Mark)

iii) Highlight the keywords that support your answer of question (ii). (4 Marks)

b) Write a paragraph using the ideas organized in the mind map shown below. Then mention the type of the paragraph. (15 Marks)

2nd Question marks: 20

a) In a project proposal, what is the difference between problem definition, motivation, and contribution? (6 Marks)

b) What is the difference between a “Resume” and a “CV”? (4 Marks)

c) In writing user guides, you should use only technical terminology. (True/False) (2 Mark)

d) Self-plagiarism is allowed in technical writing. (True/False) (2 Mark)

e) Criticize the following email subjects, (Good or Bad) stating your reasons only for the ones you find bad. (6 Marks)

i) Meeting

ii) Project Meeting Saturday, Jan. 9

iii) Meeting on Saturday to discuss the status of the graduation project before the seminar.
a) Fill in the spaces using the words/terms given below: (20 Marks)
(Copy the appended table to your answer sheet and fill in the table cells)

i. .......... is the study of the way in which certain body movements and gestures serve as a form of nonverbal communication.
ii. ........ Are movements that tend to .......... your emotions and are usually .......... 
iii. ........ Are gestures that usually replace words, and obvious one is .........
iv. ........ are gestures that accompany speech to create a visual supporting message.
v. ........ are mood indicators and are divided into three categories: ........, ........, and alter-adaptors.
vi. We have to be aware of these three essentials in order to read people accurately: ........, ........, and .........
vii. In .......... eye contact, the .......... always looks away .......... than the listener.
viii. Research shows that if you make others .......... good about themselves, through .......... and paying .......... you are perceived as .......... 

<table>
<thead>
<tr>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>vi</td>
<td>VII</td>
<td>VIII</td>
</tr>
</tbody>
</table>

b) Match the images in the first row with descriptions in the second row. (Copy the appended table and fill in the cells with the letter corresponding to your answer) (5 Marks)

<table>
<thead>
<tr>
<th>i)</th>
<th>ii)</th>
<th>iii)</th>
<th>iv)</th>
<th>v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>Open body language</td>
<td>Friendliness and honesty</td>
<td>Extreme frustration and anger</td>
<td>Offense</td>
<td>confidence and superiority</td>
</tr>
</tbody>
</table>

With My Best Wishes,
Dr. Maryam Al-Berry
Answer the following five questions: (the total marks: 60)

1. A) Choose the correct answer (10 marks)

1) \(11011 - 101\) using 1's complement equals ...........

- a) 10101
- b) 10110
- c) 00010
- d) 11101

2) If \(F(A, B, C) = (A'B' + C)\), then \(F'(A, B, C) = \ldots \ldots\)

- a) \(((A'+B).C')'\)
- b) \(((A+B').C')'\)
- c) \((A'+B).C'\)
- d) \(((A'+B)+C)\)

3) \(F(X, Y, Z) = (X+Y+Z)' + Y'Z\), if DeMorgan's law is applied to the first complemented term, then the whole expression will be ................. .

- a) \((X'+Y').(Z'+Y'Z)\)
- b) \((X+Y).Z'+Y'Z\)
- c) \(((X'+Y').Z').(Y+Z')\)
- d) \(((X'+Y').Z') + Y'Z\)

4) For a 4Mx16 RAM, the number of address lines will be equal to .......... .

- a) 12
- b) 16
- c) 22
- d) 4

5) For a 4Mx16 RAM, the size of the internal decoder is ................. .

- a) \(12\times2^{12}\)
- b) \(16\times2^{16}\)
- c) \(22\times2^{22}\)
- d) \(4\times2^{4}\)

6) For a 4Mx16 RAM, the number of input or output lines equals to .......... .

- a) 4M
- b) 16
- c) 22
- d) 8

7) To construct 8Mx32 RAM, we need ........ 4Mx16 RAMs

- a) 2
- b) 8
- c) 12
- d) 4

8) According to the above circuit, the output \(S(X, Y) = \ldots \ldots\)

- a) \(\Sigma(0, 3)\)
- b) \(\Sigma(1, 2)\)
- c) \(\Sigma(1, 3)\)
- d) \(\Sigma(0, 2)\)

9) According to the above circuit, the output \(C(X, Y) = \ldots \ldots\)

- a) \(\Sigma(0, 3)\)
- b) \(\Sigma(3)\)
- c) \(\Sigma(1)\)
- d) \(\Sigma(1, 2)\)
10) According to the above circuit, the $F(X, Y) = \ldots \ldots$

| (a) $\Sigma(1, 2, 3)$ | (b) $\Sigma(0, 1, 3)$ | (c) 0 | (d) $\Sigma(0, 3)$ |

B) Convert the following NAND circuit to AND \ OR circuit (2 marks)

![NAND Circuit Diagram]

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2nd Question (marks: 13)

A) Choose the correct answer (10 marks)

1) If the function $F(A, B, C) = \sum(0, 4, 6, 7)$ is implemented using $4 \times 1$ mux with $A$ and $C$ on selections then the inputs from $I_0$ to $I_3$ will be equal to $\ldots \ldots$ respectively.

| (a) $B', 0', B', 1$ | (b) $0, 1, 0, 1$ | (c) $B', 0, 1, B$ | (d) $B', B, B', B$ |

2) The simplest form for the Function $F(A, B, C, D) = (A+B') \cdot (A+C'+D)$ in sum of product equals $\ldots \ldots$

| (a) $A' B + A' C D'$ | (b) $A + B' D + B' C'$ |
| (c) $A + B' D + B' C' D'$ | (d) $A + A' B' D + A' B' C'$ |

3) Using the following 4 bit programmable counter with the given function table, it will count $\ldots \ldots$

![4-bit Programmable Counter Diagram]

<table>
<thead>
<tr>
<th>Clear</th>
<th>Clk</th>
<th>Load</th>
<th>Count</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>Clear all</td>
</tr>
<tr>
<td>1</td>
<td>X</td>
<td>0</td>
<td>0</td>
<td>No change</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>X</td>
<td>Load input</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>count</td>
<td></td>
</tr>
</tbody>
</table>

| (a) From 0 to 15 | (b) From 0 to 10 | (c) From 0 to 5 | (d) From 0 to 4 |

4) Using a 4 bit programmable counter with the same function table provided in the previous question, which case can give 7?
5) Given a lowest priority 4x2 encoder, in order to have a value 10 on its outputs, its inputs D0 – D3 can be equal to .............. respectively.

a) x, x, 1, 0  
b) 0, 1, x, x  
c) x, 1, 0, 0  
d) 0, 0, 1, x

B) Construct a 4 bit parallel load\ no change Register with the following function table using D flipflops and any needed external gates (3 marks).

<table>
<thead>
<tr>
<th>Select</th>
<th>Function</th>
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<tbody>
<tr>
<td>0</td>
<td>Parallel load</td>
</tr>
<tr>
<td>1</td>
<td>No change</td>
</tr>
</tbody>
</table>

3rd Question

A) For the following Decimal-Code converter table from decimal code X3 X2 X1 X0 to decimal code Y3 Y2 Y1 Y0, find the Min-terms of the outputs and don’t cares if exist. (5 marks)

<table>
<thead>
<tr>
<th>Decimal</th>
<th>X3</th>
<th>X2</th>
<th>X1</th>
<th>X0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>4</td>
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<td>1</td>
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<tr>
<td>5</td>
<td>0</td>
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<td>1</td>
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<td>8</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
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</table>

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Y3</th>
<th>Y2</th>
<th>Y1</th>
<th>Y0</th>
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<tr>
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<td>9</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
B) Construct a 8x1 multiplexer using 4x1 multiplexers and one 2x1 multiplexer.
(3 marks)

C) Follow the circuit connections, then find the function table of the given universal
shift register of 4-different functions. (All muxs have the same selection control).
(4 marks)

A) Design a counter that counts the following sequence 1, 6, 7, 3, 2 using T flipflops.
(5 marks)

B) Analyze the following sequential circuit then find state table, state diagram and
finally find the repeated sequence. Is it a self correcting counter (state your
reasons)? (8 marks)
5th Question

A) Design a sequential circuit using JK flipflop according to the following state diagram. (4 marks)

```
0\1 1\0 0\0
-\1
```

B) Find the content for the two 4 bit shift registers of the following sequential circuit during 4 clocks. [the initial value for Q is zero] (3 marks)

```
R 0 0 0 0 S
```

C) Draw the output waveform Q for the given clock signal CLK. If the frequency of the CLK is 500 HZ what will be the frequency of the output Q? [the initial value for Q is zero] (3 marks)

With My Best Regards,

Dr. Manal Tantawi
Answer the following:

**Question 1:**

a) Compare between
1) Indexing & Hashing
2) Internal & External fragmentation
3) Logical files & Physical files

b) Assume that you want to store a file of 5,00,000 records each record of size 200 bytes using Disk with the following characteristics
- Average seek time: 15 msec
- Average rotational delay: 7 msec
- Sectors per track: 70
- Transfer rate: 15 msec/track
- Tracks per cylinder: 15

How long would it take to access the entire file randomly/sequentially?

c) Calculate the expected number of overflow records for each of the following cases:
1) 1000 keys are to be stored in a hash table with 1000 addresses, no buckets used
2) 1000 keys are to be stored in a hash table with 500 addresses, bucket of size 2 used

**Question 2:**

a) What is meant by storage fragmentation? How it is happened logically in files and physically in sector-based disks? What are the ways to combat it?

b) Consider the following file containing 7 variable-length records: R1 of size 12 bytes, R2 of size 10 bytes, R3 of size 30, and R4 of size 9, R5 of size 20, R6 of size 36, and R7 of size 10

*Draw the contents of the data file and of the AVAIL LIST after all of the following steps have been completed: record R3, then R2 ; R5, R6, and R4 has been deleted, and record R8 of size 25, and record R9 of size 30, R10 of size 7, and R11 of size 10 bytes has been added (all in order), if:

1) A FIRST-FIT placement strategy has been used and procedures that can break a slot into more than one slot and then apply storage compaction
2) A WORST-FIT placement strategy has been
3) A BEST-FIT placement strategy has been used.

**c)**

1) Construct a Huffman tree for the following sentence
   SOFTWAREENGINEERINGDEPARTMENTFCISASU

2) Calculate the average number of bits per character

3) Using the Huffman Tree built in (1) encode the message “MNDWUG”.

4) Decode the message “01100011111101001”
**Question 3:**

a) What are the problems associated with secondary key indexing and hashing technique and the solutions for each.

b) Suppose we want to manage the following file of collection of second year students:

<table>
<thead>
<tr>
<th>Ref</th>
<th>Stud. ID</th>
<th>First Name</th>
<th>City</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2236789</td>
<td>Mai</td>
<td>Giza</td>
<td>Excellent</td>
</tr>
<tr>
<td>1</td>
<td>2124596</td>
<td>Sarah</td>
<td>Cairo</td>
<td>Unsufficient</td>
</tr>
<tr>
<td>2</td>
<td>3096743</td>
<td>Maya</td>
<td>Alex</td>
<td>Good</td>
</tr>
<tr>
<td>3</td>
<td>1098765</td>
<td>Ala</td>
<td>Alex</td>
<td>VGood</td>
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<td>Giza</td>
<td>Excellent</td>
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<td>2225647</td>
<td>Mostafa</td>
<td>Suez</td>
<td>VGood</td>
</tr>
<tr>
<td>12</td>
<td>1111110</td>
<td>Reda</td>
<td>Suez</td>
<td>Sufficient</td>
</tr>
<tr>
<td>13</td>
<td>1243528</td>
<td>Ahmed</td>
<td>Banha</td>
<td>Sufficient</td>
</tr>
<tr>
<td>14</td>
<td>0987643</td>
<td>Aml</td>
<td>Tanta</td>
<td>Excellent</td>
</tr>
</tbody>
</table>

1) Show the contents of the data & index files: primary using stud ID & secondary one using Grade (array & inverted list) and secondary two using City (array & inverted list)

2) Show the contents of the inverted list secondary index files after updating the following records:

<table>
<thead>
<tr>
<th>Ref</th>
<th>Stud. ID</th>
<th>First Name</th>
<th>City</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2225647</td>
<td>Mostafa</td>
<td>Cairo</td>
<td>Excellent</td>
</tr>
<tr>
<td>1</td>
<td>2231311</td>
<td>Ahmed</td>
<td>Cairo</td>
<td>Sufficient</td>
</tr>
</tbody>
</table>

**Question 4:**

a) Consider the following hash function:

```c
int hash(char *key)
{
    int h,i;
    h = position_in_the_alphabet(key[0]);
    for (i=1; i< strlen(key); i++)
        h=(i*(h + position_in_the_alphabet(key[i])))%10;
    return (h %8);
}
```

What value does the word “SOFTWARE” get hashed into?

*Note:* 1) assume key[0] = 'S', key[7] = 'E'

2) position_in_the_alphabet() function:

```c
ch: A B C D E F G H I J K L M N
position_in_the_alphabet(): 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26.
```

b) Given the following pair of keys/hash values:

<table>
<thead>
<tr>
<th>Key</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>k</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hash Value (home address)</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Hash Value (2)</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Using progressive overflow & double hashing with hash value (2) in the table above:

1) Show the contents of the hash table of size 10 containing the addresses 0-9

2) What is the effect on the average search length after deleting keys D, F, J, A, and M.
Answer the following questions: (Total marks: 65)

1st Question

1. Indicate whether the following statements are True or False and correct the false ones:
   1. An ADT (Abstract Data Type) focuses on what the structure does rather than how it is done.
   2. A singly linked list can be traversed in either direction.
   3. There are no null pointers in a singly linked list.
   4. Storing information in a linked list structure is a bad idea if we plan to do a lot of insertions.
   5. In a Queue, insertion and deletion is done at one end called top.
   6. Traversal is the process of visiting every node in a tree at least once.
   7. Every subtree of a BST (Binary Search Tree) is also a BST.
   8. A hash function should be independent of the capacity of the hash table.
   9. Rehashing requires rebuilding the hash table with a larger backing array.
   10. A graph is connected if every vertex has a path to every other vertex.

2nd Question

1. Choose the correct Answer:
   1. A node in a Linked List carries information regarding:
      a. Data  b. Link  c. Link and Data  d. None
   2. One can determine whether an infix expression has balanced parenthesis or not by using:
      a. Array  b. Queue  c. Stack  d. Tree
   3. Transform the following infix expression to postfix form: A - B / (C * D)
      a. A B C D * / -
      b. A B C D * / -
      c. D C * B A
      d. - / * A B C D
   4. To implement the queue with a linked list, keeping track of a front pointer and a back pointer. Which of these pointers will change during an insertion into an EMPTY queue?
      a. Neither changes  b. Only front changes.
      c. Only back changes.  d. Both change.
   5. One can determine whether a Binary tree is a Binary Search Tree by traversing it in:
      a. Pre-order  b. In-order  c. Post-order  d. Any of the three orders
   6. Which of the following STL classes implements a Binary Search Tree?
      a. set  b. unordered_set  c. unordered_map  d. None
   7. Which of the following statements is true?
      a. Every tree is a graph  b. A tree does not contain cycles
      c. A tree is a weakly connected graph  d. All of the above
   8. If G is a directed graph with 20 vertices, how many boolean values will be needed to represent G using an adjacency matrix?
      a. 20  b. 40  c. 200  d. 400
   9. Which of the following is useful in traversing a given graph by breadth first search?
      a. Stack  b. Set  c. List  d. Queue
10. With a poorly chosen hash function, it is possible to have a situation in which the search time in a hash table of N items goes to:
   a. \( O(1) \)  
   b. \( O(N) \)  
   c. \( O(\log N) \)  
   d. \( O(N^2) \)

3rd Question

Solve the following questions:

1. [2 marks] Show the content of a Stack \( S \) after performing the following sequence of operations:
   \( S.push(4), S.push(5), S.top(), S.push(6), S.pop(), S.push(S.Top()) \)

2. [5 marks] Given the following sequence of values:
   100, 200, 90, 125, 88, 99, 210
   a. Draw the Binary Search Tree resulting from inserting these values in the same order.
   b. Write the sequence of values resulting from the pre-order traversal of the previous tree.

3. [3 marks] Assume that you have a hash table with size 7. The hash function implemented is
   \( h(key) = key \mod 7 \). Assuming that you use separate chaining for collision resolution, show the
   contents of the hash table after inserting the following (key, value) pairs:
   (1, Star), (5, Circle), (8, Square), (3, Diamond), (11, Oval), (15, Rectangle)

4. [5 marks] Given the following graph:

   Use Dijkstra's algorithm to find the shortest path from vertex (a) to vertex (c) (Show your steps).

4th Question

Solve the following questions:

1. [8 marks] Write a C++ template class for a doubly linked list. Your implementation should
   include the Node class with its constructor and the DoublyLL class with its constructor. Add an
   implementation of the function InsertAt(v, pos) which inserts a value \( v \) into position \( pos \) in the
   list. (Note: Make sure that your function handles insertion in all possible positions)

2. [10 marks] Write a C++ template class for a Binary Search Tree. Your implementation should
   include the Node class with its constructor and the BST class with its constructor. Add an
   implementation of the functions:
   a. Find(value): which returns true if the value exists in the tree and false otherwise.
   b. Level(value): which returns the level of the tree in which the value exists (the root is at
      level 0, children of the root are at level 1, etc.)

3. [6 marks] Write a C++ method that takes two sorted queues (minimum value at the front) and
   creates a third sorted queue containing all values from both queues. Use only the queue
   operations and do not use any data structure other than the queue. (Hint: You can use the STL's
   queue class)

4. [6 marks] Suppose that you are asked to implement an undirected graph, represented by an
   adjacency list, using STL containers:
   a. What would the structure you use look like?
   b. Will the structure be different if the graph was directed? If your answer is yes, write the
      new structure.
   c. What changes would you make to include weights on the edges.

(Hint: your answer for each point should be a single line containing the declaration of the graph)

With My Best Regards,

Dr. Widad Hussein
Determine a minimum-cost shipping schedule for the transshipment problem depicted in the following figure. Find the optimal solution. Comment on the last obtained solution.

A- Consider the following linear problem

Minimize: $Z = 3x_1 + 5x_2$

Subject to:

\[x_1 \leq 4\]
\[2x_2 \leq 12\]
\[3x_1 + 2x_2 \leq 20\]

With: all variables nonnegative and integral

The optimum real Tableau for the standard form of the above problem with slack variables $S_1$, $S_2$, and $S_3$

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$S_1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-5</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$x_1$</td>
<td>-4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>$\frac{7}{6}$</td>
</tr>
</tbody>
</table>

Find the optimum integral solution using

a. Rounding method (2pts)

b. ε- close method (using one pivot only) (5pts)

B- Considering the Problem in Part A, Find the new tableau after doing the updating corresponding to each of the following cases: (don't complete to optimal solutions, only mention how you will proceed to the optimal solution (using Regular or Dual Simplex method). Solve 4 points ONLY (8pts)

i. Changing the coefficient of $x_1$ in the objective function from -4 to 1

ii. Adding new constraint $2x_1 + x_2 \leq 14$

iii. Find the range of the right-hand side of third constraint that preserves the feasibility of the solution.

iv. Changing the RHS of the first constraint from 4 to 1.

v. Adding new variable $x_k$ with $c_k=-3$ and technological coefficients $[1]$ $[1]$
A- A firm uses Lathes, milling machines and grinding machines to produce two machine parts. The following table represents the machining times required for each part, the machining times available on different machines and the profit on each machine part.

<table>
<thead>
<tr>
<th>Type of machine</th>
<th>Machining time required for the machine part (minutes)</th>
<th>Maximum time available per week (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Lathes</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>Milling machines</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Grinding machines</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Profit per unit</td>
<td>$40</td>
<td>$100</td>
</tr>
</tbody>
</table>

Find the number of parts I and II to be manufactured per week to maximize the profit. (Formulate only the mathematical program)

B- A city hospital has the following minimal daily requirement of nurses:

<table>
<thead>
<tr>
<th>Period</th>
<th>Clock time (24 hrs. day)</th>
<th>Minimal number of nurses required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 am -10 am</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>10 am -2pm</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>2 pm -6 pm</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>6 pm - 10pm</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>10 pm -2am</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>2 am -6 am</td>
<td>6</td>
</tr>
</tbody>
</table>

Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimal number of nurses to be employed so that there is sufficient number of nurses available for each period. Formulate only the linear programming problem.

C- Machineco has four jobs to be completed. Each machine must be assigned to complete one job. The time required to setup each machine for completing each job is shown in the table below. Machineco wants to minimize the total setup time needed to complete the four jobs. Find the optimal solution using Vogel's algorithm.

<table>
<thead>
<tr>
<th>Machine</th>
<th>Job1</th>
<th>Job2</th>
<th>Job3</th>
<th>Job4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine1</td>
<td>14</td>
<td>5</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Machine2</td>
<td>2</td>
<td>12</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Machine3</td>
<td>7</td>
<td>8</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Machine4</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>
A- Consider the following linear problem
Minimize: \[ Z = 9x_1 + 2x_3 + 4x_2 - x_4 \]
Subject to:
\[
\begin{align*}
5x_1 + 6x_2 + 2x_3 - 2x_4 & \geq 20 \\
-x_1 - 4x_2 + 7x_4 & \geq -40 \\
5x_1 - 4x_3 & \geq 80
\end{align*}
\]
With: all variables nonnegative.

a. Determine the symmetric dual of this program (4 points)

b. Decide which program is computationally advantageous to solve (2 points)

B- Two plants supply three customers with medical supplies. The unit costs of shipping from the plants to the customers. The production cost in plant1 is $100 and for plant2 is $110. Along with the supplies and demands, are given in the following table. The company wishes no remaining products in Plant1. The company's goal is to minimize the cost of meeting customers' demands. Find the optimal solution for this transportation problem.

<table>
<thead>
<tr>
<th>From</th>
<th>Customer1</th>
<th>Customer2</th>
<th>Customer3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant1</td>
<td>$55</td>
<td>$65</td>
<td>$80</td>
<td>35</td>
</tr>
<tr>
<td>Plant2</td>
<td>$10</td>
<td>$15</td>
<td>$25</td>
<td>50</td>
</tr>
<tr>
<td>Demand</td>
<td>10</td>
<td>20</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

With My Best Regards,
Dr. Safaa Amin